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Cambridge Assessment
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FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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1 Find the exact value of $\int_2^{\frac{7}{2}} \frac{1}{\sqrt{4x-x^2-1}} dx$.

[5]

[illegible]



2 The curve C has parametric equations

$$x = \cosh t, \quad y = \sinh t, \quad \text{for } 0 < t \leq \frac{3}{5}.$$

The length of C is denoted by s .

(a) Show that $s = \int_0^{\frac{3}{5}} \sqrt{\cosh 2t} \, dt$. [4]

[illegible]



- (b)** By finding the Maclaurin's series for $\sqrt{\cosh 2t}$ up to and including the term in t^2 , deduce an approximation to s . [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



3 The curve C has equation

$$x^3 + 2xy + 8y^3 = -12.$$

(a) Show that, at the point $(-2, -1)$ on C , $\frac{dy}{dx} = -\frac{1}{2}$. [3]

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(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-2, -1)$.

[5]

[illegible]



- (b) Use a similar method to find, in terms of N , an upper bound for $\sum_{r=1}^N \frac{1}{r^2}$. [3]

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- (c) Deduce lower and upper bounds for $\sum_{r=1}^{\infty} \frac{1}{r^2}$. [2]

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5 (a) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 25x = 338 \sin t. \quad [7]$$

[illegible]



(b) Show that, for large positive values of t and for any initial conditions,

$$x \approx R \sin(t - \phi),$$

where the constants R and ϕ are to be determined. [3]

[illegible]



- 6 (a) Show that $\sum_{r=1}^n z^{4r} = \frac{z^{4n+2} - z^2}{z^2 - z^{-2}}$, for $z^2 \neq z^{-2}$. [2]

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- (b) By letting $z = \cos \theta + i \sin \theta$, show that, if $\sin 2\theta \neq 0$,

$$\sum_{r=1}^n \sin(4r\theta) = \frac{\cos 2\theta - \cos(4n+2)\theta}{2 \sin 2\theta}. \quad [5]$$

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7 (a) Show that

$$\frac{d}{dx} \left(\frac{x}{2} \sqrt{x^2 - 9} - \frac{9}{2} \cosh^{-1} \frac{x}{3} \right) = \sqrt{x^2 - 9}. \quad [3]$$

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \sqrt{x^2 - 9},$$

given that $y = 1$ when $x = 3$. Give your answer in the form $y = f(x)$. [9]



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[illegible]



- 8** The planes Π_1 and Π_2 do not intersect and are both perpendicular to $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The line l intersects Π_1 at the point $(1, 6, 0)$ and intersects Π_2 at the point $(3, -6, 0)$.

(a) Find Cartesian equations of Π_1 and Π_2 . [3]

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(b) Express the vector equation of l in the form $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{a} + \lambda \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are vectors to be determined, and hence show that for points on l , $\frac{1}{2}x + \frac{1}{12}y = 1$ and $z = 0$. [2]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ \frac{1}{2} & \frac{1}{12} & 0 \end{pmatrix}.$$

- (c) Show that the characteristic equation of $\mathbf{A}$ is $-\lambda^3 + 3\lambda^2 + \frac{7}{4}\lambda = 0$ and hence find the eigenvalues of $\mathbf{A}$. [3]

[illegible]



(d) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^n = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$, where n is a positive integer. [6]

[illegible]



Additional page

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